## GCE

## Mathematics A

H230/01: Pure Mathematics and Statistics
Advanced Subsidiary GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

## Text Instructions

## Annotations and abbreviations

| Annotation in scoris | Meaning |
| :--- | :--- |
| $\checkmark$ and $\boldsymbol{x}$ |  |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0,1 |
| A0, A1 | Independent mark awarded 0, 1 |
| B0, B1 | Special case |
| SC | Omission sign |
| $\wedge$ | Misread |
| MR |  |
| Highlighting |  |
|  | Meaning |
| Other abbreviations in <br> mark scheme | Mark for explaining a result or establishing a given result |
| E1 | Mark dependent on a previous mark, indicated by ${ }^{*}$ |
| dep* | Correct answer only |
| cao | Or equivalent |
| oe | Rounded or truncated |
| rot | Seen or implied |
| soi | Without wrong working |
| www | Answer given |
| AG | Anything which rounds to |
| awrt | By Calculator |
| BC | This question included the instruction: In this question you must show detailed reasoning. |
| DR |  |

## Subject-specific Marking Instructions for A Level Mathematics A

Annotations should be used whenever appropriate during your marking. The $A, M$ and $B$ annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.
The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the $M$ marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, $A$ and $B$ marks are given for correct work only - differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to $\mathbf{3}$ s.f. unless the question specifically asks for another level of accuracy.
NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads " 2 s.f".
Follow through should be used so that only one mark is lost for each distinct accuracy error.
Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
If in any case the scheme operates with considerable unfairness consult your Team Leader.


| 3 |  |  | DR (all parts) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | (a) | (i) | Show $\mathrm{f}(1)=0$ | $\begin{aligned} & \text { B1 } \\ & {[1]} \end{aligned}$ | 1.1a | Correct working and result seen |  |
| 3 | (a) | (ii) | $(x-1)$ is a factor <br> Attempt find other (quadratic) factor, by any method, but must be seen $\begin{aligned} & (x-1)\left(2 x^{2}+5 x-3\right) \\ & (x-1)(2 x-1)(x+3) \\ & x=1 \text { or } \frac{1}{2} \text { or }-3 \end{aligned}$ | $\begin{gathered} \text { B1 } \\ \text { M1 } \\ \text { A1f } \\ * \\ \text { A1 } \\ {[4]} \\ \hline \end{gathered}$ | 2.2a <br> 1.1 <br> 1.1 <br> 2.2a | stated or implied, eg by attempt $\div$ by $(x-1)$ or show that $\mathrm{f}(-3)=0$ or $\mathrm{f}\left(\frac{1}{2}\right)=0$ <br> or $x=\frac{-5 \pm \sqrt{25-4 \times 2 \times(-3)}}{4}$ <br> Dep A1* cao. <br> SC correct answer no working: B1 only | or $(2 x-1)$ or $(x+3)$ is factor Inspection: Must have $2 x^{2}$ and $\pm 3$ oe <br> ft their quad factor <br> eg correct factors, no working B1 only |
| 3 | (b) |  | $\begin{aligned} & \sin \theta=1 \text { or } \sin \theta=\frac{1}{2} \\ & \theta=90^{\circ}, 30^{\circ}, 150^{\circ} \end{aligned}$ <br> $\sin \theta=-3$ gives no solution, or it doesn't exist oe or outside range or impossible or not acceptable oe | M1 <br> A3f <br> B1 [5] | $\begin{gathered} 3.2 \mathrm{a} \\ \text { 3.1a } \\ \text { 1.11 } \\ 1.1 \\ 2.3 \end{gathered}$ | Use of a root from (a). May be implied <br> Three correct, none incorrect: A3 <br> Three correct and $\geq 1$ incorrect: A2 <br> Two correct: A2 <br> One correct: A1 <br> Statement needed. Ignore all else <br> Not incorrect reason <br> eg "no solutionn because minus": B0 | ft their (a) for all A-marks <br> Ignore any "correct" solutions outside range Just "X" or "Error"not enough because DR |
| 4 | (a) |  | $\begin{aligned} & 3 x^{2}-12 x+9=0 \\ & x=3 \text { or } 1 \\ & (3,0) \text { and }(1,4) \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { A1f } \\ \text { A1 } \\ {[4]} \end{gathered}$ | $\begin{gathered} \hline 1.1 \mathrm{a} \\ 1.1 \\ 1.1 \\ 1.1 \end{gathered}$ | Attempt differentiate \& =0 May be implied Correct equation. May be implied by ans BC <br> Allow When $x=3, y=0$; when $x=1, y=4$ | $x(x-3)^{2}=0$ M1 tp at $x=3 \mathrm{~A} 1$ <br> ft their equation cao. Must be paired |
| 4 | (b) |  | Sketch (drawn in this part) of "+ve" cubic with two SPs, roughly correct shape, or just two TPs shown OR: $\mathrm{f}(1)=0$ and $\mathrm{f}(3)=0$, find $k$ for each $k>0$ or $k<-4$ | M1 <br> A1f [2] | $\begin{aligned} & \text { 3.1a } \\ & \text { 2.2a } \end{aligned}$ | Subst $x=1 \& x=3$ into $y=x^{3}-6 x^{2}+9 x \mathrm{ft}$ (a) <br> ft their (a) <br> Correct ans: M1A1 | or identify $k=-4$ and $0 . \mathrm{ft}$ (a) |




| 7 | (c) | $\begin{aligned} & 3 \mathrm{e}^{x}=1-2 \mathrm{e}^{\frac{1}{2} x} \\ & 3\left(\mathrm{e}^{\frac{1}{2} x}\right)^{2}+2 \mathrm{e}^{\frac{1}{2} x}-1=0 \quad \begin{array}{l} \text { or eg } 3 u^{2}+2 u-1=0 \\ \text { or } 3 y+2 \sqrt{y}-1=0 \end{array} \\ & \left(\left(3 \mathrm{e}^{\frac{1}{2} x}-1\right)\left(\mathrm{e}^{\frac{1}{2} x}+1\right)=0\right) \\ & \mathrm{e}^{\frac{1}{2} x}=-1: \text { no solutions stated } \\ & \mathrm{e}^{\frac{1}{2} x}=\frac{1}{3} \text { oe } \\ & x=2 \ln \frac{1}{3} \text { or } \ln \frac{1}{9} \quad \text { or }-2 \ln 3 \text { or }-\ln 9 \text { or }-2.20 \\ & y=3 \exp \left(\ln \frac{1}{9}\right)=\frac{1}{3} \quad \text { or } 0.333 \end{aligned}$ <br> Point of intersection is $\left(\ln \frac{1}{9}, \frac{1}{3}\right)$ | M1 <br> M1 <br> B1 <br> A1 <br> A1 <br> A1 <br> [6] | 3.1a <br> 3.1a <br> 2.3 <br> 2.1 <br> 1.1 <br> 2.2a | oe <br> Attempt write quadratic equation in $\mathrm{e}^{\frac{1}{2} x}$ or attempt a substitution and form QE Allow one sign error <br> May not be seen <br> eg "Cannot be negative" or "Impossible" or just "X" <br> or $\frac{1}{2} x=\ln \frac{1}{3} \quad$ or $\frac{1}{2} x=-1.10$ <br> or equivalent exact or $(-2.20,0.333)(3 \mathrm{sf})$ If any extra points of intersection: A0 | Allow decimals throughout <br> Allow substitute $x$ or $y=\mathrm{e}^{\frac{1}{2} x}$ <br> not just eg $u=\frac{1}{3}$ <br> Correct ans, with no working or irrelevant working: SC B2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | (a) | $\frac{2143}{5}$ or 428.6 or 429 <br> Height $=0.429(3 \mathrm{sf}) \quad(\mathrm{cm})$ | M1 <br> A1 <br> [2] | $\begin{gathered} 1.1 \mathrm{a} \\ 1.1 \end{gathered}$ | M1 for $\frac{2143}{4}$ or 535.75 or 534 or 0.534 Allow without "cm" | $\begin{aligned} & \text { or } \frac{2143}{4.5} \text { or } 476 \text { or } 0.476 \\ & \text { eg } \frac{1000}{428.6} \text { M1A0 } \end{aligned}$ |
| 8 | (b) | Non-numerical data oe | B1 <br> [1] | 1.2 | Indication of numerical data needed, eg Boundaries need to be between 2 numbers Cannot be measured. Qualitative data. No range of values | NOT "No class boundaries", "No class widths", "Not g'ped data", "Need continuous data" "No range" Ignore all else |
| 9 | (a) | $\begin{aligned} & \frac{\Sigma f x}{\Sigma f} \\ & =2.4 \end{aligned}$ | M1 <br> A1 [2] | $\begin{gathered} 1.1 \mathrm{a} \\ 1.1 \end{gathered}$ | attempted. May be implied by working seen or eg $\geq 2$ terms in numerator correct Dep $\div \Sigma f($ NOT $=7)$ | Allow end points or incorrect midpoints |





## Qu 11(b) Examplar solutions

A $\mathrm{H}_{0}: p=0.097$
$\mathrm{H}_{1}: p>0.097$
B1B0
$\mathrm{P}(X \geq 51)=0.138$
M1A1
$0.138>0.1$
A1
Reject $\mathrm{H}_{1}$
M1A0

B $\quad \mathrm{H}_{0}: p=0.097$
$\mathrm{H}_{1}: p>0.097$ where $p=\%$ age exp'ng side effects B1B0
$\mathrm{P}(X>51)=0.107 \quad \mathrm{M} 1 \mathrm{~A} 0$
$0.107>0.1$
A1
Not reject $\mathrm{H}_{0}$
M1A0

C $\quad \mathrm{H}_{0}: p=0.097$
$\mathrm{H}_{1}: p>0.097$ where $p=$ proportion exp'ng side effects
$\operatorname{Bin}(450,0.097)$
$\mathrm{P}(X \leq 51)=0.138$
M1A1
$0.138>0.1$
A1
Not reject $\mathrm{H}_{0}$
Under new treatment there is not a greater prop exp side effects A0

D $\quad \mathrm{H}_{0}: p=0.097$
$\mathrm{H}_{1}: p>0.097$ where $p=\mathrm{P}$ (a patient exp side effects) B1B1
$\mathrm{P}(X \leq 51)=0.108 \quad$ M1A0
$0.108>0.1$ A1
Not reject $\mathrm{H}_{0}$ M1
There is not enough evidence that the alternative hypothesis is correct (No context)

A0

E Let $p$ be the probability
$\mathrm{H}_{0}: p=0.097$
$\mathrm{H}_{1}: p>0.097$
$\operatorname{Bin}(51,0.097) \quad$ Ignore because next line is correct
$\mathrm{P}(X \leq 51)=0.138 \quad$ M1A1
$0.138>0.1$
Not reject $\mathrm{H}_{0} \quad$ M1
Under new treatment there is not a greater prop exp side effects A0

F $\quad \mathrm{H}_{0}=0.097$
$\mathrm{H}_{1}>0.097 \quad$ B0B0
$\operatorname{Bin}(450,0.1)$
$\mathrm{P}(X \geq 51)=0.192 \quad$ M0A0
$0.192>0.1$ A0
Reject $\mathrm{H}_{1}$ M1
Insuff evidence that under new treatment there is a greater prop exp side effects

A1
G $\quad \mathrm{H}_{0}: p=0.097$
$\mathrm{H}_{1}: p>0.097$ where $p=\mathrm{P}$ (a patient exp side effects) B1B1
$\operatorname{Bin}(450,0.097), \quad X=51$ or $X=50 \quad$ M1
$p=0.0308$
A0
$0.0308<0.1$
A0
Reject $\mathrm{H}_{0}$
M0
There is evidence that under new treatment there is a greater prop exp side effects

A0
$\mathrm{H}_{1}: p>0.097$
$X \sim \operatorname{Bin}(450,0.097)$
$p=0.893$
M1A0
$0.893>0.1$
Not reject $\mathrm{H}_{0}$
M0 A0

K $\mathrm{H}_{0}: p=0.097$
$\mathrm{H}_{1}: p>0.097 p$ is the likelihood of a patient $\exp$ side effects B1B1
$\mathrm{P}(X \geq 51)=0.893 \quad$ M1A0
$0.893>0.1$
A0

Not reject $\mathrm{H}_{0}$
M1
No reason to think proportion decreased
A1

L $\mathrm{H}_{0}: p=0.97$
$\mathrm{H}_{1}: p=0.97$
B0B0
$\mathrm{P}(X \geq 50)=0.0366$
M1A0
$0.0366<0.1$
A0

Reject $\mathrm{H}_{0}$ There is evidence prop has increased

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